Entangled state for constructing generalized phase space representation and its statistical behavior

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Abstract

Based on the conception of quantum entanglement of Einstein-Podolsky-Rosen we construct generalized phase space representation associated with the entangled state $|\Gamma\rangle_e$, which is endowed with definite physical meaning. The set of states make up a complete and non-orthogonal representation. The Weyl ordered form of $|\Gamma\rangle_{ee}\langle\Gamma|$ is derived which clearly exhibit the statistical behavior of marginal distribution of $|\Gamma\rangle_{ee}\langle\Gamma|$. The minimum uncertainty relation obeyed by $|\Gamma\rangle_e$ is also demonstrated.

1 Introduction

Phase space formalism of quantum mechanics has many applications in quantum statistics, quantum optics and quantum information theory. It began with Wigner's celebrated paper in 1932 [1]. Among many kinds of pseudo-probability distribution functions, Wigner function $W(q,p)$ of a quantum state (pure or mixed states) is the most popularly used, since in phase space it exhibits two marginal distribution as the following way [2],

$$\mathrm{P}(p)=\int_{-\infty}^{\infty}W(q,p)dq,\ \mathrm{P}\left(q\right)=\int_{-\infty}^{\infty}W(q,p)dp, \tag{1}$$

where $\mathrm{P}\left(q\right)$ $[\mathrm{P}(p)]$ is proportional to the probability for finding the particle at q [at p in momentum space]. Besides, the Wigner operator also serves as an integral kernel of the Weyl rule [3, 4] which is a quantization scheme connecting classical functions of (q,p) with their quantum correspondence operators of (Q,P). The single-mode Wigner operator in the coordinate representation is

$$\Delta(q,p)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|q-\frac{v}{2}\right\rangle\left\langle q+\frac{v}{2}\right|e^{-ipv}dv, \tag{2}$$

where $|q\rangle$ is the coordinate eigenvector, $Q\left|q\right\rangle=q\left|q\right\rangle,$

$$|q\rangle=\pi^{-1/4}\exp\left[-\frac{q^2}{2}+\sqrt{2}qa^{\dagger}-\frac{1}{2}a^{\dagger 2}\right]|0\rangle\,. \tag{3}$$

Using the normally ordered form of vacuum projector $|0\rangle\langle 0| =: \exp[-a^{\dagger}a]:$, where $:\ :$ denotes normal ordering, and the technique of integration within an ordered product (IWOP) of operators [5, 6], the integration in Eq. (2) can be performed, leading to the explicit operator

$$\Delta(q,p)=\frac{1}{\pi}:e^{-(q-Q)^2-(p-P)^2}:\ =\frac{1}{\pi}:e^{-2(a^{\dagger}-\alpha^*)(a-\alpha)}:\ \equiv\Delta(\bar{\alpha},\bar{\alpha}^*), \tag{4}$$

where Q and P are related to the Bose creation and annihilation operators $\left(a^{\dagger},a\right)$ by $Q=(a+a^{\dagger})/\sqrt{2}$ and $P=(a-a^{\dagger})/(i\sqrt{2})$, respectively, $[a,a^{\dagger}]=1$, $\bar{\alpha}=(q+ip)/\sqrt{2}$. Obviously, $\int_{-\infty}^{\infty}\Delta(q,p)dp=\frac{1}{\sqrt{\pi}}\colon e^{-(q-Q)^2}\colon=|q\rangle\langle q|$, $\int_{-\infty}^{\infty}\Delta(q,p)dq=\frac{1}{\sqrt{\pi}}\colon e^{-(p-P)^2}\colon=|p\rangle\langle p|$, where $|p\rangle$ is the momentum eigenvector. This formalism helps us to understand the Wigner function's role better[2][7]-[10].

In two-mode case when two systems are prepared in an entangled state, measuring one of the two canonically conjugate variables on one system, the value for a physical variable in the another system may be inferred with certainty, this is the quantum entanglement. Because entanglement is now widely used in quantum information and quantum computation, it has been paid much attention by physicists [11]. The original idea of quantum entanglement began with Einstein-Podolsky-Rosen's observation in EPR's treatment [12], who noticed that two particles' relative coordinate Q_1-Q_2 and total momentum P_1+P_2 can be simultaneously measured, (also their conjugate variables $[Q_1+Q_2,P_1-P_2]=0$), therefore, the corresponding Wigner function should be such that its two marginal distributions are respectively proportional to the probability for finding the two particles which possess certain total momentum value [relative momentum value] and simultaneously relative position value [center-of-mass position value] (see also Eqs. (9) and (12)). The investigation of Wigner functions for entangled states is not only just for the convenience of some calculations, but also for revealing the intrinsic entanglement property inherent to some physical systems. In Ref. [13] by virtue of the well-behaved properties of the entangled state representation $\langle\eta|$,

$$|\eta\rangle=\exp\left\{-\frac{1}{2}|\eta|^2+\eta a_1^{\dagger}-\eta^* a_2^{\dagger}+a_1^{\dagger}a_2^{\dagger}\right\}|00\rangle,\ \eta=\eta_1+i\eta_2, \tag{5}$$

we have successfully established the so-called entangled Wigner operator for correlated two-body systems [13],

$$\Delta_w(\rho,\varsigma)=\int\frac{d^2\eta}{\pi^3}|\rho-\eta\rangle\langle\rho+\eta|\exp(\eta\varsigma^*-\eta^*\varsigma), \tag{6}$$

$|\eta\rangle$ is the common eigenvector of Q_1-Q_2 and P_1+P_2 [14], which obeys the eigenvector equations

$$(Q_1-Q_2)|\eta\rangle=\sqrt{2}\eta_1|\eta\rangle,\ \ (P_1+P_2)|\eta\rangle=\sqrt{2}\eta_2|\eta\rangle. \tag{7}$$

Using the IWOP technique we have shown in **[13]** that $\Delta_w(\rho,\varsigma)$ is just the product of two independent single-mode Wigner operators $\Delta_w(\rho,\varsigma)=\Delta(\bar{\alpha},\bar{\alpha}^*)\Delta(\bar{\beta},\bar{\beta}^*)$ provided we take

$$\varsigma=\bar{\alpha}+\bar{\beta}^*,\ \rho=\bar{\alpha}-\bar{\beta}^*,\ \ \bar{\alpha}=\frac{q_1+ip_1}{\sqrt{2}},\ \bar{\beta}=\frac{q_2+ip_2}{\sqrt{2}}. \tag{8}$$

Performing the integration of $\Delta_w(\rho,\varsigma)$ over $d^2\varsigma$ leads to the projection operator of the entangled state $|\eta\rangle$

$$\int d^2\varsigma\Delta_w(\rho,\varsigma)=\frac{1}{\pi}|\eta\rangle\langle\eta|\,|_{\eta=\rho}, \tag{9}$$

and the marginal distribution in (η_1,η_2) phase space is $\langle\psi|\int d^2\varsigma\Delta_w(\rho,\varsigma)|\psi\rangle=\frac{1}{\pi}|\psi(\eta)|^2|_{\eta=\rho}$.(in reference to (7)). Similarly, we can introduce the common eigenvector of Q_1+Q_2 and P_1-P_2 [14] (the conjugate state of $|\eta\rangle$)

$$|\xi\rangle=\exp\left\{-\frac{1}{2}|\xi|^2+\xi a_1^{\dagger}+\xi^* a_2^{\dagger}-a_1^{\dagger}a_2^{\dagger}\right\}|00\rangle,\ \ \xi=\xi_1+i\xi_2, \tag{10}$$

which obeys another pair of eigenvector equations

$$(Q_1+Q_2)|\xi\rangle=\sqrt{2}\xi_1|\xi\rangle,\ \ (P_1-P_2)|\xi\rangle=\sqrt{2}\xi_2|\xi\rangle. \tag{11}$$

Performing the integration of $\Delta_w(\rho,\varsigma)$ over $d^2\rho$ yields

$$\int d^2\rho\Delta_w(\rho,\varsigma)=\frac{1}{\pi}|\xi\rangle\langle\xi|\,|_{\xi=\varsigma},\ \langle\psi|\int d^2\rho\Delta_w(\rho,\varsigma)|\psi\rangle=\frac{1}{\pi}|\psi(\xi)|^2|_{\xi=\varsigma}. \tag{12}$$

The introduction of the entangled Wigner operator also brings much convenience for calculating the Wigner function of some entangled states.

Working in the $|\eta\rangle$ *or* $|\xi\rangle$ representation one can interrelate some physical systems. For example, the Einstein-Podolsky-Rosen arrangement relies on free propagation of quantum-coupled particles (described by $|\eta\rangle$ or $|\xi\rangle$), on the other hand, the two-mode squeezed state dealing with oscillators which are bound systems, these two seemingly very different physical systems can be interrelated by constructing the following ket-bra integration in terms of $|\eta\rangle$,

$$\int \frac{d^2\eta}{\pi\mu}|\frac{\eta}{\mu}\rangle\langle\eta| = \exp\left[\left(a_1^\dagger a_2^\dagger - a_1 a_2\right)\ln\mu\right], \tag{13}$$

the right hand-side of (13) is just the two-mode squeezing operator [7, 15]. In the following we shall employ the $|\eta\rangle$ state to formulating generalized phase space representation and then study its statistical behaviors, i.e. enlightened by Eqs. (7) and (11) we construct generalized phase space representation characteristic of the properties under the action of $Q_1 - Q_2$ and $P_1 + P_2$, associated with a two-mode state vector $|\Gamma\rangle_e$,

$$\begin{aligned} {}_e\langle\Gamma| \frac{Q_1 - Q_2}{\sqrt{2}} &= \left(\alpha\sigma_1 + i\beta\frac{\partial}{\partial\tau_2}\right) {}_e\langle\Gamma|, \\ {}_e\langle\Gamma| \frac{P_1 - P_2}{\sqrt{2}} &= \left(\gamma\tau_2 + i\delta\frac{\partial}{\partial\sigma_1}\right) {}_e\langle\Gamma|, \end{aligned} \tag{14}$$

where the subscript "e" implies the entanglement, α, β, γ and δ are all real parameters, satisfying

$$\beta\gamma - \alpha\delta = 1, \tag{15}$$

and $[Q_1 - Q_2, P_1 - P_2] = 2i$. Simultaneously, under the action of the center-of-mass operator and the relative momentum operator, the state ${}_e\langle\Gamma|$ behaves

$$\begin{aligned} {}_e\langle\Gamma| \frac{Q_1 + Q_2}{\sqrt{2}} &= \left(\gamma\tau_1 - i\delta\frac{\partial}{\partial\sigma_2}\right) {}_e\langle\Gamma|, \\ {}_e\langle\Gamma| \frac{P_1 + P_2}{\sqrt{2}} &= \left(\alpha\sigma_2 - i\beta\frac{\partial}{\partial\tau_1}\right) {}_e\langle\Gamma|. \end{aligned} \tag{16}$$

The present paper is arranged as follows. In Sec. 2 using the newly developed bipartite entangled state representation $|\eta\rangle$ of continuum variables we shall derive the concrete form of entangled state $|\Gamma\rangle_e$ in two-mode Fock space and then analyze its properties, in so doing the phase space theory can be developed to the entangled case. In Sec. 3, the completeness relation and non-orthonormal property of $|\Gamma\rangle_e$ are proved. In Sec. 4. the Weyl ordered form of $|\Gamma\rangle_{ee}\langle\Gamma|$ is derived, which yields the classical correspondence of $|\Gamma\rangle_{ee}\langle\Gamma|$. In Sec. 5 we examine marginal distributions of the operator $|\Gamma\rangle_{ee}\langle\Gamma|$ by using the properties of the entangled state $|\eta\rangle$ and its conjugate state $|\xi\rangle$. The uncertainty relation of coordinate and momentum quadratures in $|\Gamma\rangle_e$ and the Wigner function of $|\Gamma\rangle_e$ are calculated in sections 6 and 7, respectively.

2 The state $|\Gamma\rangle_e$ in two-mode Fock space

We find that the explicit form of the state $|\Gamma\rangle_e$ in two-mode Fock space is (see the Appendix),

$$|\Gamma\rangle_e \equiv 2\sqrt{-\alpha\beta\gamma\delta}\exp\left[\frac{\alpha|\sigma|^2}{2\delta} - \frac{\gamma|\tau|^2}{2\beta} + (\alpha\sigma + \gamma\tau)a_1^\dagger + (\gamma\tau^* - \alpha\sigma^*)a_2^\dagger - (\beta\gamma + \alpha\delta)a_1^\dagger a_2^\dagger\right]|00\rangle, \tag{17}$$

where $\sigma = \sigma_1 + i\sigma_2$, $\tau = \tau_1 + i\tau_2$; real numbers (α, β, γ and δ) satisfy the relation Eq.(15); $(a_i, a_i^\dagger)$, $i = 1, 2$, are the two-mode Bose annihilation and creation operators obeying $\left[a_i, a_j^\dagger\right] = \delta_{ij}$. To

satisfy the square integrable condition for wave function in phase space $|\Gamma\rangle_e$, $\frac{\alpha}{\delta}<0$ and $\frac{\gamma}{\beta}>0$ are demanded. In order to certify that Eq.(17) really obeys Eqs. (14) and (16) we operate a_i on $|\Gamma\rangle_e$,

$$\begin{aligned} a_1|\Gamma\rangle_e &= \left[(\alpha\sigma+\gamma\tau)-(\beta\gamma+\alpha\delta)\,a_2^\dagger\right]|\Gamma\rangle_e\,, \\ a_2|\Gamma\rangle_e &= \left[(\gamma\tau^*-\alpha\sigma^*)-(\beta\gamma+\alpha\delta)\,a_1^\dagger\right]|\Gamma\rangle_e\,. \end{aligned} \tag{18}$$

Then noting the relation between Q_i, P_j and $a_i, a_j^\dagger$,

$$Q_i=(a_i+a_i^\dagger)/\sqrt{2},\ P_i=(a_i-a_i^\dagger)/(\sqrt{2}\mathtt{i}), \tag{19}$$

and Eq.(15) as well as

$$\begin{aligned} \frac{\partial}{\partial\sigma}\,{}_e\langle\Gamma| &= {}_e\langle\Gamma|\left(\frac{\alpha\sigma^*}{2\delta}-\alpha a_2\right),\ \frac{\partial}{\partial\sigma^*}\,{}_e\langle\Gamma| = {}_e\langle\Gamma|\left(\frac{\alpha\sigma}{2\delta}+\alpha a_1\right), \\ \frac{\partial}{\partial\tau}\,{}_e\langle\Gamma| &= {}_e\langle\Gamma|\left(-\frac{\gamma\tau^*}{2\beta}+\gamma a_2\right),\ \frac{\partial}{\partial\tau^*}\,{}_e\langle\Gamma| = {}_e\langle\Gamma|\left(-\frac{\gamma\tau}{2\beta}+\gamma a_1\right), \end{aligned} \tag{20}$$

we see, for example,

$$\begin{aligned} {}_e\langle\Gamma|\frac{Q_1+Q_2}{\sqrt{2}} &= {}_e\langle\Gamma|\left[-\delta\left(\alpha a_1+\alpha a_2\right)-i\alpha\sigma_2+\gamma\tau_1\right] \\ &= \left[\gamma\tau_1+\delta\left(\frac{\partial}{\partial\sigma}-\frac{\partial}{\partial\sigma^*}\right)\right]{}_e\langle\Gamma| \\ &= \left(\gamma\tau_1-i\delta\frac{\partial}{\partial\sigma_2}\right){}_e\langle\Gamma|\,, \end{aligned} \tag{21}$$

which is the first equation in Eq.(16). In a similar way, ${}_e\langle\Gamma|$ satisfying the other equations in Eqs.(14) and (16) can be checked. Using Eqs.(14), (16) and noticing the quantum commutator

$$\left[\frac{Q_1\pm Q_2}{\sqrt{2}},\frac{P_1\pm P_2}{\sqrt{2}}\right]=i, \tag{22}$$

in the phase space representation, we have

$${}_e\langle\Gamma|\left[\frac{Q_1\pm Q_2}{\sqrt{2}},\frac{P_1\pm P_2}{\sqrt{2}}\right]=i\left(\beta\gamma-\alpha\delta\right){}_e\langle\Gamma|\,, \tag{23}$$

which results in the condition shown in Eq.(15).

3 The properties of $|\Gamma\rangle_e$

3.1 The completeness relation of $|\Gamma\rangle_e$

Next we prove the completeness relation of Eq.(17). Using the normally ordered vacuum projector

$$|00\rangle\langle 00|=:\exp\left(-a_1^\dagger a_1-a_2^\dagger a_2\right):, \tag{24}$$

where $:\ :$ denotes the normal product, which means all the bosonic creation operators are standing on the left of annihilation operators in a monomial of $a^\dagger$ and a [16]. It should be emphasized that a normally ordered product of operators can be integrated with respect to c-numbers provided the

integration is convergent. Then we can use Eq.(17) and the IWOP technique to perform the following integration

$$\begin{aligned}
&\frac{1}{\beta^2\delta^2}\int\frac{d^2\sigma d^2\tau}{4\pi^2}\left|\Gamma\right\rangle_{ee}\left\langle\Gamma\right| \\
=\ & -\frac{\alpha\gamma}{\beta\delta}\int\frac{d^2\sigma d^2\tau}{\pi^2}\colon\exp\left[\frac{\alpha\left|\sigma\right|^2}{\delta}+\sigma\alpha\left(a_1^\dagger-a_2\right)+\sigma^*\alpha\left(a_1-a_2^\dagger\right)-a_1^\dagger a_1\right. \\
& \left.-\frac{\gamma\left|\tau\right|^2}{\beta}+\tau\gamma\left(a_1^\dagger+a_2\right)+\tau^*\gamma\left(a_2^\dagger+a_1\right)-\left(\beta\gamma+\alpha\delta\right)\left(a_1^\dagger a_2^\dagger+a_1a_2\right)-a_2^\dagger a_2\right]\colon \\
=\ & -\frac{\alpha\gamma}{\beta\delta}\int\frac{d^2\sigma d^2\tau}{\pi^2}\colon\exp\left\{\frac{\alpha}{\delta}\left[\sigma+\delta\left(a_1-a_2^\dagger\right)\right]\left[\sigma^*+\delta\left(a_1^\dagger-a_2\right)\right]\right. \\
& \left.-\frac{\gamma}{\beta}\left[\tau-\beta\left(a_2^\dagger+a_1\right)\right]\left[\tau^*-\beta\left(a_1^\dagger+a_2\right)\right]\right\}\colon \\
=\ & \colon\exp\left[-\left(a_1^\dagger a_1+a_2^\dagger a_2\right)\left(\alpha\delta-\beta\gamma+1\right)\right]\colon=1,
\end{aligned}\tag{25}$$

where we have used the integral formula [17]

$$\int\frac{d^2\beta}{\pi}\exp\left[\varsigma\left|\beta\right|^2+\xi\beta+\eta\beta^*\right]=-\frac{1}{\varsigma}\exp\left[-\frac{\xi\eta}{\varsigma}\right],\ \mathrm{Re}\varsigma<0.\tag{26}$$

Thus $\left|\Gamma\right\rangle_e$ is capable of making up a new quantum mechanical representation.

3.2 The non-orthonormal property of $\left|\Gamma\right\rangle_e$

Noticing the overlap relation

$$\begin{aligned}
{}_e\left\langle\Gamma\right|z_1,z_2\rangle &= 2\sqrt{-\alpha\beta\gamma\delta}\exp\left[-\frac{\left|z_1\right|^2}{2}+\frac{\alpha\left|\sigma\right|^2}{2\delta}-\frac{\gamma\left|\tau\right|^2}{2\beta}+\left(\alpha\sigma^*+\gamma\tau^*\right)z_1\right] \\
&\quad\times\exp\left[-\frac{\left|z_2\right|^2}{2}+\left(\gamma\tau-\alpha\sigma\right)z_2-\left(\beta\gamma+\alpha\delta\right)z_1z_2\right],
\end{aligned}\tag{27}$$

where $\left|z\right\rangle=\exp\left(-\left|z\right|^2/2+za^\dagger\right)\left|0\right\rangle$ is the coherent state [18, 19] and using the over-completeness relation of coherent states $\int\frac{d^2z_1d^2z_2}{\pi^2}\left|z_1,z_2\right\rangle\left\langle z_1,z_2\right|=1$, we can derive the inner-product ${}_e\left\langle\Gamma\right|\Gamma'\rangle_e$, ($\left|\Gamma'\right\rangle_e$ has the same β,γ,α and δ with $\left|\Gamma\right\rangle_e$),

$$\begin{aligned}
{}_e\left\langle\Gamma\right|\Gamma'\rangle_e &= \int\frac{d^2z_1d^2z_2}{\pi^2}\,{}_e\left\langle\Gamma\right|z_1,z_2\rangle\left\langle z_1,z_2\right|\Gamma'\rangle_e \\
&= -4\alpha\beta\gamma\delta\int\frac{d^2z_1d^2z_2}{\pi^2}\exp\left[-\left|z_1\right|^2+\left(\alpha\sigma^*+\gamma\tau^*\right)z_1+\left(\alpha\sigma'+\gamma\tau'\right)z_1^*\right. \\
&\quad-\left|z_2\right|^2+\left(\gamma\tau-\alpha\sigma\right)z_2+\left(\gamma\tau'^*-\alpha\sigma'^*\right)z_2^*-\left(\beta\gamma+\alpha\delta\right)z_1z_2 \\
&\quad\left.+\frac{\alpha}{2\delta}\left(\left|\sigma\right|^2+\left|\sigma'\right|^2\right)-\frac{\gamma}{2\beta}\left(\left|\tau\right|^2+\left|\tau'\right|^2\right)-\left(\beta\gamma+\alpha\delta\right)z_1^*z_2^*\right].
\end{aligned}\tag{28}$$

With the aid of the integral formula Eq.(26), we perform the integral over $d^2z_1d^2z_2$ in Eq.(28) and finally obtain

$$\begin{aligned}
{}_e\left\langle\Gamma\right|\Gamma'\rangle_e &= \exp\left[\frac{\alpha}{4\beta\gamma\delta}\left|\sigma-\sigma'\right|^2-\frac{1}{4\beta\delta}\left(\tau'\sigma^*-\sigma\tau'^*+\sigma'\tau^*-\tau\sigma'^*\right)\right. \\
&\quad\left.+\frac{\gamma}{4\alpha\beta\delta}\left|\tau-\tau'\right|^2-\frac{\left(\beta\gamma+\alpha\delta\right)}{4\beta\delta}\left(\tau'\sigma'^*-\sigma'\tau'^*+\sigma\tau^*-\tau\sigma^*\right)\right].
\end{aligned}\tag{29}$$

From Eq.(29) one can see that ${}_e\langle\Gamma|\,\Gamma'\rangle_e$ is non-orthogonal, only when $\sigma=\sigma'$ and $\tau=\tau'$, ${}_e\langle\Gamma|\,\Gamma\rangle_e=1$.

4 The Weyl ordered form of $|\Gamma\rangle_{ee}\langle\Gamma|$

For a density operator ρ of bipartite system, we can convert it into its Weyl ordered form [3, 4, 20] by using the formula

$$\rho=4\int\frac{d^2z_1d^2z_2}{\pi^2}\,\vdots\,\langle -z_1,-z_2|\,\rho\,|z_1,z_2\rangle\exp\left[2\sum_{i=1}^{2}\left(a_i^\dagger a_i+a_iz_i^*-z_ia_i^\dagger\right)\right]\vdots, \tag{30}$$

where the symbol $\vdots\ \vdots$ denotes the Weyl ordering, $|z_i\rangle$ is the coherent state, $\langle -z_i|\,z_i\rangle=\exp\{-2|z_i|^2\}$. Note that the order of Bose operators a_i and $a_i^\dagger$ within a Weyl ordered product can be permuted. That is to say, even though $\left[a,a^\dagger\right]=1$, we can have $\vdots\,aa^\dagger\,\vdots=\vdots\,a^\dagger a\,\vdots$. Substituting Eq.(17) into Eq.(30) and performing the integration by virtue of the technique of integration within a Weyl ordered product (IWWOP) of operators [21], we finally obtain

$$\begin{aligned}
|\Gamma\rangle_{ee}\langle\Gamma| &= -16\alpha\beta\gamma\delta\int\frac{d^2z_1d^2z_2}{\pi^2}\,\vdots\,\exp\Big[-|z_1|^2+\left(\sigma^*\alpha+\tau^*\gamma-2a_1^\dagger\right)z_1+\left(2a_1-\sigma\alpha-\tau\gamma\right)z_1^*\\
&\quad -|z_2|^2+\left(\tau\gamma-\sigma\alpha-2a_2^\dagger\right)z_2+\left(2a_2-\tau^*\gamma+\sigma^*\alpha\right)z_2^*\\
&\quad -\left(\beta\gamma+\alpha\delta\right)\left(z_1^*z_2^*+z_1z_2\right)+\frac{\alpha|\sigma|^2}{\delta}-\frac{\gamma|\tau|^2}{\beta}+2a_1^\dagger a_1+2a_2^\dagger a_2\Big]\vdots\\
&= 4\,\vdots\exp\left\{\frac{\alpha\delta}{\beta\gamma}\left(\frac{\sigma}{\delta}+\left(a_1-a_2^\dagger\right)\right)\left(\frac{\sigma^*}{\delta}+\left(a_1^\dagger-a_2\right)\right)\right.\\
&\quad\left.+\frac{\gamma\beta}{\alpha\delta}\left(\frac{\tau}{\beta}-\left(a_1+a_2^\dagger\right)\right)\left(\frac{\tau^*}{\beta}-\left(a_2+a_1^\dagger\right)\right)\right\}\vdots,
\end{aligned}\tag{31}$$

or

$$\begin{aligned}
|\Gamma\rangle_{ee}\langle\Gamma| &= 4\,\vdots\exp\left\{\frac{\alpha\delta}{\beta\gamma}\left[\left(\frac{\sigma_1}{\delta}+\frac{Q_1-Q_2}{\sqrt{2}}\right)^2+\left(\frac{\sigma_2}{\delta}+\frac{P_1+P_2}{\sqrt{2}}\right)^2\right]\right.\\
&\quad\left.+\frac{\beta\gamma}{\alpha\delta}\left[\left(\frac{\tau_1}{\beta}-\frac{Q_1+Q_2}{\sqrt{2}}\right)^2+\left(\frac{\tau_2}{\beta}-\frac{P_1-P_2}{\sqrt{2}}\right)^2\right]\right\}\vdots,
\end{aligned}\tag{32}$$

which is the Weyl ordered form of $|\Gamma\rangle_{ee}\langle\Gamma|$. Noting the difference between Eq.(31) and Eq.(25), they are in different operator ordering. The merit of Weyl ordering lies in the Weyl ordered operators' invariance under similar transformations, which was proved in Ref.[22]. In addition, it is very convenient for us to obtain the marginal distributions of $|\Gamma\rangle_{ee}\langle\Gamma|$ (see the next section).

In Ref. [23] we have derived the Weyl ordering form of two-mode Wigner operator $\Delta_w(\rho;\varsigma)$

$$\Delta_w(\rho;\varsigma)=\vdots\,\delta\left(a_1-a_2^\dagger-\rho\right)\delta\left(a_1^\dagger-a_2-\rho^*\right)\delta\left(a_1+a_2^\dagger-\varsigma\right)\delta\left(a_1^\dagger+a_2-\varsigma^*\right)\vdots. \tag{33}$$

Eq.(33) indicates that the Weyl quantization scheme, for bipartite entangled operator, is to take the following correspondence,

$$\rho\to\left(a_1-a_2^\dagger\right),\ \varsigma\to\left(a_1+a_2^\dagger\right), \tag{34}$$

then the form of Eq.(31) indicates that the classical Weyl function corresponding to $|\Gamma\rangle_{ee}\langle\Gamma|$ is

$$4\exp\left[\frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta}+\rho\right|^2+\frac{\gamma\beta}{\alpha\delta}\left|\frac{\tau}{\beta}-\varsigma\right|^2\right]\equiv h(\rho;\varsigma). \tag{35}$$

Thus the Weyl quantization rule in this case is embodied as

$$
\begin{aligned}
|\Gamma\rangle_{ee}\langle\Gamma| &= 4\int d^2\rho d^2\varsigma \,\vdots\, \delta\left(a_1 - a_2^\dagger - \rho\right)\delta\left(a_1^\dagger - a_2 - \rho^*\right)\delta\left(a_1 + a_2^\dagger - \varsigma\right) \\
&\quad \times\delta\left(a_1^\dagger + a_2 - \varsigma^*\right)\,\vdots\, \exp\left[\frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta}+\rho\right|^2 + \frac{\gamma\beta}{\alpha\delta}\left|\frac{\tau}{\beta}-\varsigma\right|^2\right] \\
&= 4\int d^2\rho d^2\varsigma \Delta_w(\rho;\varsigma)\exp\left[\frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta}+\rho\right|^2 + \frac{\gamma\beta}{\alpha\delta}\left|\frac{\tau}{\beta}-\varsigma\right|^2\right]. \qquad (36)
\end{aligned}
$$

Using the IWOP technique and Eq.(24), in [13] we have shown that the normally ordered form of $\Delta_w(\rho;\varsigma)$ is

$$
\Delta_w(\rho;\varsigma) = \frac{1}{\pi^2}\colon \exp\left[-\left(a_1 - a_2^\dagger - \rho\right)\left(a_1^\dagger - a_2 - \rho^*\right) - \left(a_1 + a_2^\dagger - \varsigma\right)\left(a_1^\dagger + a_2 - \varsigma^*\right)\right]\colon. \qquad (37)
$$

Substituting Eq.(37) into Eq.(36) yields

$$
\begin{aligned}
|\Gamma\rangle_{ee}\langle\Gamma| &= 4\int \frac{d^2\rho d^2\varsigma}{\pi^2}\colon \exp\left\{-\left(a_1 - a_2^\dagger - \rho\right)\left(a_1^\dagger - a_2 - \rho^*\right) + \frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta}+\rho\right|^2\right. \\
&\quad \left. -\left(a_1 + a_2^\dagger - \varsigma\right)\left(a_1^\dagger + a_2 - \varsigma^*\right) + \frac{\gamma\beta}{\alpha\delta}\left|\frac{\tau}{\beta}-\varsigma\right|^2\right\}\colon \\
&= -4\alpha\beta\gamma\delta\colon \exp\left\{\frac{\alpha}{\delta}\left[\sigma + \delta\left(a_1 - a_2^\dagger\right)\right]\left[\sigma^* + \delta\left(a_1^\dagger - a_2\right)\right]\right. \\
&\quad \left. -\frac{\gamma}{\beta}\left[\tau - \beta\left(a_2^\dagger + a_1\right)\right]\left[\tau^* - \beta\left(a_1^\dagger + a_2\right)\right]\right\}\colon, \qquad (38)
\end{aligned}
$$

which confirms Eq.(25). In particular, when $\beta = -\delta = 1$, and $\alpha = \frac{\kappa}{1+\kappa}$, $\gamma = \frac{1}{1+\kappa}$, Eq.(36) becomes

$$
|\Gamma\rangle_{ee}\langle\Gamma| \to 4\int d^2\rho d^2\varsigma \Delta_w(\rho,\varsigma)\exp\left[-\kappa|\rho-\sigma|^2 - \frac{1}{\kappa}|\varsigma-\tau|^2\right], \qquad (39)
$$

which is the generalization of single-mode Husimi operator [24, 25].

5 Marginal distributions of $|\Gamma\rangle_{ee}\langle\Gamma|$

As mentioned above, based on the Weyl ordered form Eq.(32) it is very convenient for us to obtain the marginal distributions of $|\Gamma\rangle_{ee}\langle\Gamma|$,

$$
\int_{-\infty}^{\infty}\frac{d^2\sigma}{\pi}|\Gamma\rangle_{ee}\langle\Gamma| = -\frac{4\beta\gamma\delta}{\alpha}\,\vdots\,\exp\left\{\frac{\beta\gamma}{\alpha\delta}\left[\left(\frac{\tau_1}{\beta} - \frac{Q_1+Q_2}{\sqrt{2}}\right)^2 + \left(\frac{\tau_2}{\beta} - \frac{P_1-P_2}{\sqrt{2}}\right)^2\right]\right\}\vdots. \qquad (40)
$$

Noting $[Q_1+Q_2, P_1-P_2] = 0$, there is no operator ordering problem involved in Eq.(40), so the symbol $\vdots\ \vdots$ in Eq.(40) can be neglected,

$$
\int_{-\infty}^{\infty}\frac{d^2\sigma}{\pi}|\Gamma\rangle_{ee}\langle\Gamma| = -\frac{4\beta\gamma\delta}{\alpha}\exp\left\{\frac{\beta\gamma}{\alpha\delta}\left[\left(\frac{\tau_1}{\beta} - \frac{Q_1+Q_2}{\sqrt{2}}\right)^2 + \left(\frac{\tau_2}{\beta} - \frac{P_1-P_2}{\sqrt{2}}\right)^2\right]\right\}. \qquad (41)
$$

The completeness relation of $|\xi\rangle$ expressed in Eq.(10) is

$$
\begin{aligned}
\int \frac{d^2\xi}{\pi}|\xi\rangle\langle\xi| &= 1,\ d^2\xi = d\xi_1 d\xi_2, \qquad (42) \\
\langle\xi'|\xi\rangle &= \pi\delta(\xi'-\xi)\delta(\xi'^*-\xi^*), \qquad (43)
\end{aligned}
$$

we see that the marginal distribution of function $|_e \langle \Gamma | \Psi \rangle|^2$ in "ξ-direction" is given by

$$\begin{aligned} \langle \Psi | \int_{-\infty}^{\infty} \frac{d^2\sigma}{\pi} |\Gamma\rangle_{ee} \langle \Gamma | \Psi \rangle &= \langle \Psi | \int \frac{d^2\xi}{\pi} |\xi\rangle \langle \xi | \int_{-\infty}^{\infty} \frac{d^2\sigma}{\pi} |\Gamma\rangle_{ee} \langle \Gamma \int \frac{d^2\xi'}{\pi} |\xi'\rangle \langle \xi' | \Psi \rangle \\ &= -\frac{4\beta\gamma\delta}{\alpha} \int_{-\infty}^{\infty} \frac{d^2\xi}{\pi} |\Psi(\xi)|^2 \exp\left[\frac{\beta\gamma}{\alpha\delta}\left|\frac{\tau}{\beta} - \xi\right|^2\right], \end{aligned} \tag{44}$$

which is a Gaussian-broadened version of quantal distribution $|\Psi(\xi)|^2$ (measuring two particles' relative momentum and center-of-mass coordinate). Similarly, we can obtain another marginal distribution by performing the integral $d^2\tau$ over $|\Gamma\rangle_{ee}\langle\Gamma|$,

$$\int_{-\infty}^{\infty} \frac{d^2\tau}{\pi} |\Gamma\rangle_{ee} \langle \Gamma| = -\frac{4\alpha\beta\delta}{\gamma} \exp\left\{\frac{\alpha\delta}{\beta\gamma}\left[\left(\frac{\sigma_1}{\delta} + \frac{Q_1 - Q_2}{\sqrt{2}}\right)^2 + \left(\frac{\sigma_2}{\delta} + \frac{P_1 + P_2}{\sqrt{2}}\right)^2\right]\right\}. \tag{45}$$

By using the completeness relation of $|\eta\rangle$,

$$\begin{aligned} \int \frac{d^2\eta}{\pi} |\eta\rangle\langle\eta| &= 1, \quad d^2\eta = d\eta_1 d\eta_2, \end{aligned} \tag{46}$$

$$\langle \eta' | \eta \rangle = \pi\delta(\eta' - \eta\xi)\,\delta(\eta'^* - \eta^*), \tag{47}$$

we see that the other marginal distribution of $|_e \langle \Gamma | \Psi \rangle|^2$ in "η-direction" is

$$\langle \Psi | \int_{-\infty}^{\infty} \frac{d^2\tau}{\pi} |\Gamma\rangle_{ee} \langle \Gamma | \Psi \rangle = -\frac{4\alpha\beta\delta}{\gamma} \int_{-\infty}^{\infty} \frac{d^2\eta}{\pi} |\Psi(\eta)|^2 \exp\left\{\frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta} + \eta\right|^2\right\}, \tag{48}$$

a Gaussian-broadened version of quantal distribution $|\Psi(\eta)|^2$ (measuring two particles' relative coordinate and total momentum), Eqs.(44) and (48) describe the relationship between wave functions in the $_e\langle\Gamma|$ representation and those in EPR entangled state $|\xi\rangle$ ($|\eta\rangle$) representation, respectively. Note that $|\eta\rangle$ and $|\xi\rangle$ are related to each other by

$$\langle \xi | \eta \rangle = \frac{1}{2} \exp\left(\frac{\xi^*\eta - \xi\eta^*}{2}\right). \tag{49}$$

6 Minimum uncertainty relation for $|\Gamma\rangle_e$

From the marginal distributions of $|\Gamma\rangle_{ee}\langle\Gamma|$ we have seen that its phase space representation involves both the center-of mass (relative) coordinate and the relative (total) momentum. In order to see clearly how the state $|\Gamma\rangle_e$ obeys uncertainty relation, we introduce two pairs of quadrature phase amplitudes for two-mode field:

$$Q_\pm \equiv \frac{Q_1 \pm Q_2}{\sqrt{2}}, \; P_\pm \equiv \frac{P_1 \pm P_2}{\sqrt{2}}, \; [Q_\pm, P_\pm] = i. \tag{50}$$

In similar to deriving Eq.(29), using Eqs. (5), (10) and (17), we calculate the overlap between $\langle\eta|$ and $|\Gamma\rangle_e$,

$$\langle \eta | \Gamma \rangle_e = \sqrt{-\frac{\alpha\delta}{\beta\gamma}} \exp\left\{\frac{\alpha\delta}{2\beta\gamma}\left|\frac{\sigma}{\delta} + \eta\right|^2 + \frac{1}{2\beta}\left[\tau(\eta^* - \alpha\sigma^*) - \tau^*(\eta - \alpha\sigma)\right]\right\}, \tag{51}$$

and the overlap between $\langle\xi|$ and $|\Gamma\rangle_e$,

$$\langle \xi | \Gamma \rangle_e = \sqrt{-\frac{\beta\gamma}{\alpha\delta}} \exp\left\{\frac{\beta\gamma}{2\alpha\delta}\left|\frac{\tau}{\beta} - \xi\right|^2 - \frac{1}{2\delta}\left[\sigma(\xi^* - \gamma\tau^*) - \sigma^*(\xi - \gamma\tau)\right]\right\}. \tag{52}$$

Then employing the completeness relation of $|\eta\rangle$ and Eq.(51), we evaluate

$$\begin{aligned} \langle Q_-\rangle &= \int \frac{d^2\eta}{\pi}\eta_1 \left|\langle \eta \left|\Gamma\right\rangle_e\right|^2 = -\frac{\sigma_1}{\delta}, \\ \left\langle Q_-^2\right\rangle &= \int \frac{d^2\eta}{\pi}\eta_1^2 \left|\langle \eta \left|\Gamma\right\rangle_e\right|^2 = \frac{\sigma_1^2}{\delta^2} - \frac{\beta\gamma}{2\alpha\delta}, \end{aligned} \tag{53}$$

and

$$\langle P_-\rangle = \frac{\tau_2}{\beta},\ \left\langle P_-^2\right\rangle = \frac{\tau_2^2}{\beta^2} - \frac{\alpha\delta}{2\beta\gamma}. \tag{54}$$

It then follows

$$\begin{aligned} \left\langle \Delta Q_-^2\right\rangle &= \left\langle Q_-^2\right\rangle - \langle Q_-\rangle^2 = -\frac{\beta\gamma}{2\alpha\delta}, \\ \left\langle \Delta P_-^2\right\rangle &= \left\langle P_-^2\right\rangle - \langle P_-\rangle^2 = -\frac{\alpha\delta}{2\beta\gamma}, \end{aligned} \tag{55}$$

and

$$\sqrt{\left\langle \Delta Q_-^2\right\rangle \left\langle \Delta P_-^2\right\rangle} = \frac{1}{2}. \tag{56}$$

In a similar way, using (52) we can derive

$$\begin{aligned} \langle Q_+\rangle &= \frac{\sigma_2}{\delta},\ \left\langle Q_+^2\right\rangle = \frac{\sigma_2^2}{\delta^2} - \frac{\beta\gamma}{2\alpha\delta}, \\ \langle P_+\rangle &= \frac{\tau_1}{\beta},\ \left\langle P_+^2\right\rangle = \frac{\tau_1^2}{\beta^2} - \frac{\alpha\delta}{2\beta\gamma}, \end{aligned} \tag{57}$$

which also leads to

$$\sqrt{\left\langle \Delta Q_+^2\right\rangle \left\langle \Delta P_+^2\right\rangle} = \frac{1}{2}. \tag{58}$$

Eqs. (55)-(58) show that $|\Gamma\rangle_e$ is a minimum uncertainty state for the two pairs of quadrature operators.

7 The Wigner function of $|\Gamma\rangle_e$

For a bipartite system, the two-mode Wigner operator in entangled state $|\eta\rangle$ representation is expressed in Eq.(6), so the Wigner function $W(\rho,\varsigma)$ of $|\Gamma\rangle_e$ is given by

$$W(\rho,\varsigma) = Tr\left[|\Gamma\rangle_{ee}\langle\Gamma|\,\Delta_w(\rho,\varsigma)\right] = \int \frac{d^2\eta}{\pi^3}\,{}_e\langle\Gamma\left|\rho-\eta\right\rangle\left\langle\rho+\eta\right|\Gamma\rangle_e\, e^{\eta\varsigma^*-\varsigma\eta^*}. \tag{59}$$

Substituting Eq.(51) into Eq.(59) and using the formula in Eq.(26), we obtain

$$\begin{aligned} W(\rho,\varsigma) &= \int \frac{d^2\eta}{\pi^3}\,{}_e\langle\Gamma\left|\rho-\eta\right\rangle\left\langle\rho+\eta\right|\Gamma\rangle_e\, e^{\eta\varsigma^*-\varsigma\eta^*} \\ &= -\frac{\alpha\delta}{\beta\gamma}\int\frac{d^2\eta}{\pi^3}\exp\left\{\frac{\alpha\delta}{\beta\gamma}|\eta|^2 + \left(\varsigma^*-\frac{\tau^*}{\beta}\right)\eta + \left(\frac{\tau}{\beta}-\varsigma\right)\eta^* \right. \\ &\quad \left. +\frac{\alpha}{2\beta\gamma\delta}\left(2|\sigma|^2 + 2\delta^2|\rho|^2 + 2\delta\left(\sigma\rho^*+\rho\sigma^*\right)\right)\right\} \\ &= \frac{1}{\pi^2}\exp\left[\frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta}+\rho\right|^2 + \frac{\gamma\beta}{\alpha\delta}\left|\frac{\tau}{\beta}-\varsigma\right|^2\right]. \end{aligned} \tag{60}$$

From Eq.(60) one can see that the Wigner function of $|\Gamma\rangle_e$ possesses the well-behaved feature in the sense that its marginal distribution in "σ-direction" is a general Gaussian form $\exp\left\{\frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta}+\rho\right|^2\right\}$,

while its marginal distribution in "τ-direction" is $\exp\left\{\frac{\beta\gamma}{\alpha\delta}\left|\frac{\tau}{\beta}-\varsigma\right|^{2}\right\}$. When $\beta\gamma+\alpha\delta=0$ and $\beta=-\delta=1$, Eq.(60) reduces to

$$W\left(\rho,\varsigma\right)=\frac{1}{\pi^{2}}\exp\left(-\left|\sigma+\rho\right|^{2}-\left|\tau-\varsigma\right|^{2}\right), \tag{61}$$

which is just the Wigner function of two-mode canonical coherent state.

Before ending this work, we mention that in 1990 Torres-Vega and Frederick introduced the state $\left|\Gamma\right\rangle$ which satisfies [30]

$$\begin{aligned}\left\langle\Gamma\right|Q_{1} &= \left(\alpha q+i\beta\frac{\partial}{\partial p}\right)\left\langle\Gamma\right|, &(62)\\ \left\langle\Gamma\right|P_{1} &= \left(\gamma p+i\delta\frac{\partial}{\partial q}\right)\left\langle\Gamma\right|. &(63)\end{aligned}$$

these two equations well satisfy the correspondence between the classical and quantum Liouville equations in single-mode case and have many applications in chemical physics and quantum chemistry [30]-[34]. Recently $\left|\Gamma\right\rangle$ has been identified as a coherent squeezed state [35]

$$\left|\Gamma\right\rangle\equiv\left(2\sqrt{-\alpha\beta\gamma\delta}\right)^{1/2}\exp\left[\frac{\alpha q^{2}}{2\delta}-\frac{\gamma p^{2}}{2\beta}+\sqrt{2}\left(\alpha q+i\gamma p\right)a_{1}^{\dagger}+\frac{\beta\gamma+\alpha\delta}{2}a_{1}^{\dagger 2}\right]\left|0\right\rangle_{1}. \tag{64}$$

The $\left|\Gamma\right\rangle_{e}$ in this paper is the non-trivial generalization of $\left|\Gamma\right\rangle$.

In summary, based on the conception of quantum entanglement of Einstein-Podolsky-Rosen, we have introduced the entangled state $\left|\Gamma\right\rangle_{e}$ for constructing generalized phase space representation, which possesses well-behaved properties. The set of $\left|\Gamma\right\rangle_{e}$ make up a complete and non-orthogonal representation, so it may have new applications, for examples: 1) It can be chosen as a good representation for solving dynamic problems for some Hamiltonians which include explicitly the function of quadrature operators $Q_{\pm}$, and/or $P_{\pm}$; 2) $\left|\Gamma\right\rangle_{ee}\left\langle\Gamma\right|$ may be considered as a generalized Wigner operator, since from Eq. (36) we see that it is expressed as smoothing out the usual Wigner operator by averaging over a "course graining" function $\exp\left[\frac{\alpha\delta}{\beta\gamma}\left|\frac{\sigma}{\delta}+\rho\right|^{2}+\frac{\gamma\beta}{\alpha\delta}\left|\frac{\tau}{\beta}-\varsigma\right|^{2}\right]$, and the corresponding generalized Wigner function is positive definite. 3) The representation $\left|\Gamma\right\rangle_{e}$ may be used to analyze entanglement degree for some entangled states. **4)** The $\left|\Gamma\right\rangle_{e}$ state can be taken as a quantum channel for quantum teleportation, such channel may make the teleportation fidelity flexible, since it involves adjustable parameters $\alpha,\beta,\gamma,\delta$. We hope these applications could be studied in the near future.

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APPENDIX Derivation of Eq.(17)

Here we show how to derive Eq.(17). In the $\left|\eta\right\rangle$ representation we have

$$\begin{aligned}\frac{Q_{1}+Q_{2}}{\sqrt{2}}\left|\eta\right\rangle &= -i\frac{\partial}{\partial\eta_{2}}\left|\eta\right\rangle=\left(\frac{\partial}{\partial\eta}-\frac{\partial}{\partial\eta^{*}}\right)\left|\eta\right\rangle, &(A1)\\ \frac{P_{1}-P_{2}}{\sqrt{2}}\left|\eta\right\rangle &= i\frac{\partial}{\partial\eta_{1}}\left|\eta\right\rangle=i\left(\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\eta^{*}}\right)\left|\eta\right\rangle, &(A2)\end{aligned}$$

so according to the requirement in Eqs. (14)-(16), we see

$$\begin{aligned}\left[\frac{\gamma}{2}\left(\tau+\tau^{*}\right)+\delta\left(\frac{\partial}{\partial\sigma}-\frac{\partial}{\partial\sigma^{*}}\right)\right]{}_{e}\left\langle\Gamma\right|\left.\eta\right\rangle &= \left(\frac{\partial}{\partial\eta}-\frac{\partial}{\partial\eta^{*}}\right){}_{e}\left\langle\Gamma\right|\left.\eta\right\rangle, &(A3)\\ \left[\frac{\alpha}{2i}\left(\sigma-\sigma^{*}\right)-i\beta\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\tau^{*}}\right)\right]{}_{e}\left\langle\Gamma\right|\left.\eta\right\rangle &= \frac{\eta-\eta^{*}}{2i}{}_{e}\left\langle\Gamma\right|\left.\eta\right\rangle, &(A4)\end{aligned}$$

and

$$\left[\frac{\alpha}{2}\left(\sigma+\sigma^*\right)-\beta\left(\frac{\partial}{\partial\tau}-\frac{\partial}{\partial\tau^*}\right)\right]{}_e\left\langle\Gamma\right|\left.\eta\right\rangle=\frac{\eta+\eta^*}{2}{}_e\left\langle\Gamma\right|\left.\eta\right\rangle, \tag{A5}$$

$$\left[\frac{\gamma}{2i}\left(\tau-\tau^*\right)+i\delta\left(\frac{\partial}{\partial\sigma}+\frac{\partial}{\partial\sigma^*}\right)\right]{}_e\left\langle\Gamma\right|\left.\eta\right\rangle=i\left(\frac{\partial}{\partial\eta}+\frac{\partial}{\partial\eta^*}\right){}_e\left\langle\Gamma\right|\left.\eta\right\rangle, \tag{A6}$$

Combining Eqs.(A3)-(A6) yields

$$\left(\alpha\sigma+2\beta\frac{\partial}{\partial\tau^*}\right){}_e\left\langle\Gamma\right|\left.\eta\right\rangle=\eta\,{}_e\left\langle\Gamma\right|\left.\eta\right\rangle, \tag{A7}$$

$$\left(\alpha\sigma^*-2\beta\frac{\partial}{\partial\tau}\right){}_e\left\langle\Gamma\right|\left.\eta\right\rangle=\eta^*\,{}_e\left\langle\Gamma\right|\left.\eta\right\rangle, \tag{A8}$$

$$\left(\gamma\tau^*+2\delta\frac{\partial}{\partial\sigma}\right){}_e\left\langle\Gamma\right|\left.\eta\right\rangle=2\frac{\partial}{\partial\eta}\,{}_e\left\langle\Gamma\right|\left.\eta\right\rangle \tag{A9}$$

$$\left(\gamma\tau-2\delta\frac{\partial}{\partial\sigma^*}\right){}_e\left\langle\Gamma\right|\left.\eta\right\rangle=-2\frac{\partial}{\partial\eta^*}\,{}_e\left\langle\Gamma\right|\left.\eta\right\rangle. \tag{A10}$$

The solution to Eqs.(A7)-(A10) is

$${}_e\left\langle\Gamma\right|\left.\eta\right\rangle=C\exp\left\{\frac{\alpha\delta}{2\beta\gamma}\left|\frac{\sigma}{\delta}+\eta\right|^2+\frac{1}{2\beta}\left[\tau^*\left(\eta-\alpha\sigma\right)-\tau\left(\eta^*-\alpha\sigma^*\right)\right]\right\}, \tag{A11}$$

where C is the normalization constant determined by ${}_e\left\langle\Gamma\right|\left.\Gamma\right\rangle_e=1$.

Using the completeness relation of EPR entangled state (46) and the integral formula in Eq.(26), we obtain

$$\begin{aligned}{}_e\left\langle\Gamma\right|&=\int\frac{d^2\eta}{\pi}{}_e\left\langle\Gamma\right|\left.\eta\right\rangle\left\langle\eta\right|\\&=C\left\langle00\right|\int\frac{d^2\eta}{\pi}\exp\left[-\frac{1}{2}|\eta|^2+\eta^*a_1-\eta a_2+a_1a_2\right]\\&\times\exp\left\{\frac{\alpha}{2\beta\gamma\delta}\left|\sigma+\delta\eta\right|^2+\frac{1}{2\beta}\left[\tau^*\left(\eta-\alpha\sigma\right)-\tau\left(\eta^*-\alpha\sigma^*\right)\right]\right\}\\&=\left\langle00\right|C\exp\left[\frac{\alpha\left|\sigma\right|^2}{2\delta}-\frac{\gamma\left|\tau\right|^2}{2\beta}+\left(\alpha\sigma^*+\gamma\tau^*\right)a_1+\left(\gamma\tau-\alpha\sigma\right)a_2-\left(\beta\gamma+\alpha\delta\right)a_1a_2\right],\end{aligned} \tag{A12}$$

which is just Eq.(17) when C is taken as $C=2\sqrt{-\alpha\beta\gamma\delta}$.

References

[1] Wigner E P 1932 Phys. Rev. **40** 749

[2] Feynman R P 1972 *StatisticalMechanics* (New York: Benjamin)

[3] Weyl H 1927 Z. Phys. **46** 1

[4] Weyl H 1953 *The Classical Groups* (Princeton University Press)

[5] Fan H Y, Lu H L and Fan Y 2006 Ann. Phys. **321** 480; Fan H Y 2003 J. Opt. B: Quantum Semiclass. Opt. **5** R147

[6] Wünsche A 1999 J. Opt. B: Quantum Semiclass. Opt. **1** R11

[7] Walls D F and Milburn G 1994 *Quantum Optics* (Berlin:Springer)

[8] Smithey D T, Beck M and Raymer M G 1993 Phys. Rev. Lett. **70** 1244

[9] Wu H J and Fan H Y 1997 Mod. Phys. Lett. B **11** 544; Fan H Y 2001 J. Opt. B: Quantum Semiclass. Opt. **3** 388; Fan H Y and Sun Z H 2000 Phys. Lett. A **272** 219

[10] Vogel K and Risken H 1989 Phys. Rev. A **40** 2847; Tatarsky T 1983 Sov. Phys.–Usp. **26** 311

[11] Bouwmeester D *et al* 2000 *The Physics of Quantum Information* (Springer, Berlin);

Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge University Press).

[12] Einstein A, Podolsky B and Rosen N 1935 Phys. Rev. **47** 777

[13] Fan H Y 2002 Phys. Rev. A. **65** 064102; Wu H J and Fan H Y 1997 Mod. Phys. Lett. B **11** 549

[14] Fan H Y and Klauder J R 1994 Phys. Rev. A. **47** 704; Fan H Y and Chen B Z 1996 Phys. Rev. A. **53** 2948; Fan H Y and Ye X 1995 Phys. Rev A. **51** 3343

[15] Bužek V 1990 J. Mod. Opt. **37** 303;

Loudon R and Knight P L 1987 J. Mod. Opt. **34** 709;

Fan H Y and Fan Y, 1996 Phys. Rev. A. **54** 958;

Dodonov V V 2002 J. Opt. B: Quant. Semiclass. Opt. **4** R1

[16] Louisell W H 1973 *Quantum Statistical Properties of Radiation* (New York: Wiley)

[17] Puri R R 2000 *Mathematical Methods of Quantum Optics* (Springer-Verlag, Berlin), Appendix A.

[18] Glauber R J 1963 Phys. Rev. **130** 2529; 1963 Phys. Rev. **131** 2766

[19] Klauder J R and Skargerstam B S 1985 *Coherent States* (*World Scientific*, Singapore)

[20] Fan H Y 1992 J. Phys. A, **25** 3443

[21] Fan H Y 2007 Ann. Phys. doi: 10.1016/j.aop.2007.08.009

[22] Fan H Y 2008 Ann. Phys. **323** 500

[23] Fan H Y, Fan Y 2002 Int. J. Mod. Phys. A, **17** 701; Fan H Y 2000 Mod. Phys. Lett. A **15** 2297

[24] Husimi K 1940 Proc. Phys. Math. Soc. Jpn. **22** 264

[25] Fan H Y and Yang Y L 2006. Phys. Lett. A **353** 439

[26] Wolfgang P. Schleich 2001 *Quantum Optics in Phase Space* (Wiley-VCH, Birlin);

Dodonov V V, Man'ko V I 2003 *Theory of Nonclassical States of Light* (Taylor & Francis, New York)

[27] Agawal G S, Wolf E 1972 Phys. Rev. D **2** 2161;

Agawal G S, Wolf E 1972 Phys. Rev. D **2** 2187;

Agawal G S, Wolf E 1972 Phys. Rev. D **2** 2206

[28] Bužek V, Knight P L 1995 Prog. Opt. **34** 1;

Bužek V, Keitel C H, Knight P L 1995 Phys. Rev. A **51** 2575

[29] O'Connell R F, Wigner E P 1981 Phys. Lett. A **83** 145;

Hillery M, O'Connell R F, Scully M O and Wigner E P 1984 Phys. Rep. **106** 121;

Vogel K, Risken H 1989 Phys. Rev. A **40** 2847

[30] Go. Torres-Vega and Frederick J H 1993 J. Chem. Phys. **98** 3103

[31] Go. Torres-Vega and Frederick J H 1990 J. Chem. Phys. **93**, 8862

[32] Bopp F 1961 *in Werner Heisenberg und die Physik unserer Zeit* (Vieweg, Braunschweig)

[33] Jaffé C and Brumer P 1985 J. Chem. Phys. **82** 2330;
Jaffé C, Kanfer S and Brumer P 1985 Phys. Rev. Lett. **54** 8

[34] Davis M J 1988 J. Phys. Chem. **92** 3124; Gray S K 1987 J. Chem. Phys. **87** 2051; Klaus B. Møller, Thomas G. Jørgensen and Go. Torres-Vega 1997 J. Phys. Chem. **106** 7228

[35] Hu L Y, Fan H Y and Lu H L 2008 J. Chem. Phys. **128** 054101